Name ____ Test 3, Spring 2021

1) Find all the eigenspaces of the matrix below (15 points)

$$\begin{bmatrix} -1 & -3 & 3 \\ 0 & -1 & 0 \\ 0 & -3 & 2 \end{bmatrix}$$

$$|xI - A| = \begin{vmatrix} x+1 & 3 & -3 \\ 0 & x+1 & 0 \\ 0 & 3 & x-2 \end{vmatrix} = (x+1)\begin{vmatrix} x+1 & 0 \\ 3 & x-2 \end{vmatrix} = (x+1)(x+1)(x-2)$$

For $\lambda = -1$ we get:

$$\begin{bmatrix} -1+1 & 3 & -3 \\ 0 & -1+1 & 0 \\ 0 & 3 & -1-2 \end{bmatrix} = \begin{bmatrix} 0 & 3 & -3 \\ 0 & 0 & 0 \\ 0 & 3 & -3 \end{bmatrix} \sim_R \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This has eigenspace $span\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right\}$.

For $\lambda = 2$ we get:

$$\begin{bmatrix} 2+1 & 3 & -3 \\ 0 & 2+1 & 0 \\ 0 & 3 & 2-2 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -3 \\ 0 & 3 & 0 \\ 0 & 3 & 0 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This has eigenspace $span\left\{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right\}$.

2) Find the diagonalizaion of the matrix from the previous problem. (5 points)
(If you couldn't solve the previous problem, make up an answer to answer this problem)

$$\begin{bmatrix} -1 & -3 & 3 \\ 0 & -1 & 0 \\ 0 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}^{-1}$$

3) Given the basis below, find an orthogonal basis for the same vector space. (10 points)

$$\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}\right\}$$

The first vector is $\begin{bmatrix}1\\1\\0\end{bmatrix}$

The second vector is $\begin{bmatrix}2\\3\\4\end{bmatrix} - proj_{\begin{bmatrix}1\\1\\0\end{bmatrix}}\left(\begin{bmatrix}2\\3\\4\end{bmatrix}\right)$

$$proj_{\begin{bmatrix}1\\1\\0\end{bmatrix}}\left(\begin{bmatrix}2\\3\\4\end{bmatrix}\right) = \frac{2+3}{2}\begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}2.5\\2.5\\0\end{bmatrix}$$

$$\begin{bmatrix}2\\3\\4\end{bmatrix} - \begin{bmatrix}2.5\\2.5\\0\end{bmatrix} = \begin{bmatrix}-0.5\\0.5\\4\end{bmatrix}$$

The orthogonal basis is:

$$\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}-0.5\\0.5\\4\end{bmatrix}\right\}$$

4) Answer the following questions. (3 points each)

A) Let A be a 5×5 with eigenvalues $0, 0, 1, 2, 3$. What is the maximum rank of A?

4

B) Let A be a 3×5 matrix whose nullity is 4. When row reduced, how many rows of zeroes are there?

2

C) Consider a system of 4 equations and 4 variables that has a unique solution. When row reduced, how many pivots does the corresponding matrix have?

4

D) Let A be a 6×6 matrix whose corresponding linear transformation T is onto. Is T one-to-one?

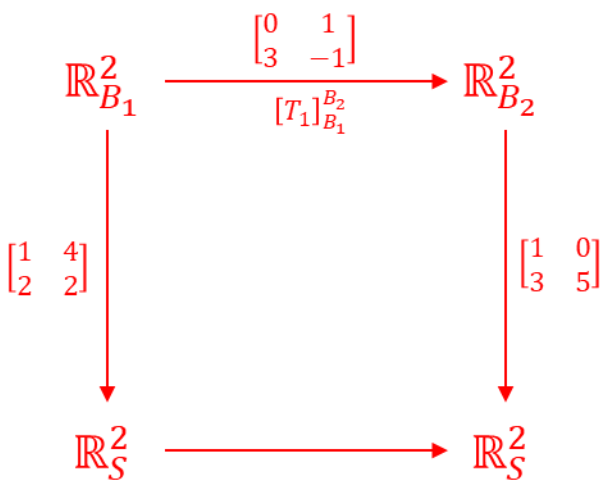
Yes

E) Let A be a 3×3 matrix whose corresponding linear transformation T is not one-to-one. What is the determinant of A?

0

5) Given the two bases and linear transformation below, draw the diagram that represents this information. (10 points)

$$B_1 = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \end{bmatrix} \right\}, B_2 = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \end{bmatrix} \right\}, T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{B_1} \right) = \begin{bmatrix} x_2 \\ 3x_1 - x_2 \end{bmatrix}_{B_2}$$



6) Find the product below. (10 points)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 7 \\ -6 & 19 \end{bmatrix}$$

7) Reduce the matrix below to reduced echelon form. (10 points)

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 6 \\ 0 & 1 & 3 & 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 6 \\ 0 & 1 & 3 & 4 & 1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 6 \\ 0 & 1 & 3 & 4 & 1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 4 & 1 \\ 0 & 0 & 1 & 2 & 6 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & -17 \\ 0 & 0 & 1 & 2 & 6 \end{bmatrix}$$

8) Find the product below. (5 points)

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 & 2 & 2 \\ 3 & 1 & 3 & 3 & 3 \\ 4 & 4 & 1 & 4 & 4 \\ 5 & 5 & 5 & 1 & 5 \\ 6 & 6 & 6 & 6 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 & -2 & -2 & -2 \\ 3 & 1 & 3 & 3 & 3 \\ 14 & 14 & 11 & 6 & 14 \\ 5 & 5 & 5 & 1 & 5 \\ 6 & 6 & 6 & 6 & 1 \end{bmatrix}$$

9) Find the length of the vector below. (5 points)

$$\begin{bmatrix} 3 \\ 4 \\ 0 \\ 12 \end{bmatrix}$$

$$\sqrt{3^2 + 4^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

10) Given system of equations below, write the corresponding matrix equation. (5 points)

$$x + 2y = 3$$
$$4x + 5y = 6$$

$$\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

11) Find a formula for the pseudoinverse of the matrix below. (5 points)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$$\left(\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

12) Given the information below about three bases B_1, B_2, B_3, and two linear transformaitons T_1 and T_2, find a formula for $[T_1]_{B_1}^{B_2}$. (5 points)

$$B_1 = \left\{ \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right\}$$
$$B_2 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \end{bmatrix} \right\}$$
$$B_3 = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$$
$$[T_2 \circ T_1]_S^S = \left\{ \begin{bmatrix} 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$$
$$[T_2]_{B_2}^{B_3} = \left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 5 & 2 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$